

CHAPTER ELEVEN

SET

A set is defined as a collection of items

The Number System:

1. Our number system can be divided into the following group of set of numbers.

(1) The Set of integers i.e. $\{\dots -3, -2, -1, 0, 1, 2, 3, \dots\}$.

– Integers refer to negative and positive whole numbers as well as zero.

2) The set of whole numbers i.e. $\{0, 1, 2, 3, 4, \dots\}$.

– Whole numbers are numbers which are greater than zero, including zero itself.

3) The set of natural or counting numbers i.e. $\{1, 2, 3, 4, 5, \dots\}$.

– Natural numbers are number from 1 upwards.

4) The set of odd numbers i.e. $\{1, 3, 5, 7, 9, \dots\}$.

- Odd numbers are those numbers, which when divided by 2 always give us a remainder or a decimal, but 1 is an odd number.

5) The set of prime numbers i.e. $\{2, 3, 5, 7, 11, 13, 17, \dots\}$.

– Prime numbers are those numbers which have only two factors. Since 7 has two factors which are 1 and 7, then it is a prime number.

– On the other hand, $9 = 3 \times 3$ and $9 = 1 \times 9 \Rightarrow 9$ has four factors, which are 3 and 3, as well as 1 and 9. For this reason it is not a prime number.

6) The set of composite numbers i.e. $\{4, 6, 8, 9, 10, \dots\}$.

– These are numbers which have two or more factors apart from itself and 1. For example apart from 1 and 20, 20 which is a composite number has four other factors which are $\{4, 5\}$ and $\{2, 10\}$.

– Also apart from 1 and 6, 6 which is a composite number has two other factors which are 2 and 3.

7) The set of even numbers i.e. $\{2, 4, 6, 8, 10, 12, \dots\}$.

– These are those numbers, which can be divided by 2 without a remainder or a decimal.

8) The set of irrational numbers i.e.

$\{\dots \pi, \sqrt{3}, \sqrt{5}, \frac{1}{3}, \frac{2}{6}, \dots\}$.

– This consists of square roots of numbers which does not give us whole numbers, as well as fractions without specific values. For example $\frac{1}{3} = 0.333333 \dots$ and $\frac{2}{3} = 0.6666 \dots$

- Lastly π or pie, even though taken to be $= \frac{22}{7}$ or 3.14, really has no fixed value.

10) Set of real numbers i.e. $\{\dots -3, -2, -1, 0, 1, 2, 3.5, \sqrt{7} \dots\}$, which consists of all the various sets just discussed.

FACTORS OF A GIVEN NUMBER:

- These are whole numbers which can divide that given number, without any remainder, with the given number being the highest factor. Examples are; (1) The factors of 6 = 1, 2, 3, 6 (2) The factors of 8 = 1, 2, 4, 8 (3) The factors of 30 = 1, 2, 3, 5, 6, 15, 30.

MULTIPLE OF A GIVEN NUMBER:

- If y is our number, then the multiples of $y = 1 \times y, 2 \times y, 3 \times y, 4 \times y \dots = y, 2y, 3y, 4y$; For example, the multiples of 2 = $2 \times 1, 2 \times 2, 2 \times 3, 2 \times 4, 2 \times 5 \dots = 2, 4, 6, 8, 10 \dots$ Also the multiples of 5 = $5 \times 1, 5 \times 2, 5 \times 3, 5 \times 4 \dots = 5, 10, 15, 20, 25 \dots$

Q1. Find the set of natural numbers from 1 to 12.

Soln

The set of natural numbers from 1 to 12 = $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ or $\{1, 2, 3, 4, \dots, 12\}$.

Q2. Find the set of the even natural numbers from 1 to 12.

NB: First find the set of natural numbers from 1 to 12, and select the even ones among them.

Soln

$\Rightarrow \{\text{Natural numbers from 1 to 12}\} = \{1, 2, 3, \dots, 12\}$.

$\Rightarrow \{\text{Even natural numbers}\} = \{2, 4, 6, 8, 10, 12\}$.

Q3. Determine the set of the multiples of 3, which are less than 15.

Soln

$\{\text{Multiples of 3 less than 15}\} = \{3, 6, 9, 12\}$.

Q4. Find the set of the odd multiples of 3 up to 18.

Soln

The multiples of 3 up to 18 = {3,6,9,12,15,18} and select the odd ones among them $\Rightarrow$ { Odd multiples of 3 upto 18} = {3, 9, 15}.

The Number of Elements:

The number of elements of a set A is written as $n(A)$.

Therefore if $A = \{a,b,c\}$, then $n(A) = 3$ and also if $Y = \{1,2\}$, then $n(Y) = 2$.

Types of Sets:

There are various types of sets and these are :

1. A Finite set:

- This is a set whose members can be counted, and an example is the set of people within a family.

2. An Infinite set:

- This is a set which contains an uncountable number of items or elements.

- An example is the set of the number of buckets of water that can be fetched from the sea.

3. Equal Sets:

- If $A = \{1,2,3\}$ and $B = \{2,3,1\}$, then A and B are said to be equal sets.

– Two sets are said to be equal, if they contain the same elements or items, no matter the order or manner in which they have been arranged.

- Also if $Z = \{a,b,c,d\}$ and $X = \{b,a,c,d\}$, then Z and X are equal sets.

4. Equivalent sets:

- These are two sets, in which the number of items or elements found in each is the same.

- For example if $X = \{a,b\}$ and $Y = \{1,2\}$, then X and Y are equivalent sets.

5. The Null Set:

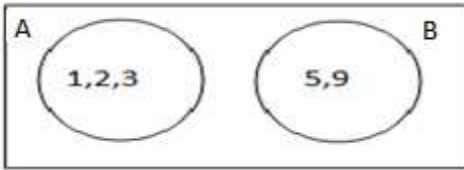
- This is a set which has no members, and it is represented by the symbol $\{ \}$ or $\emptyset$.

– For example $\{\text{People who live in the sea}\} = \emptyset$.

6. Disjointed Sets:

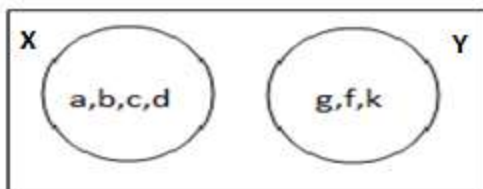
- These are two sets which do not contain any element in common.

– Example 1. If $A = \{1,2,3\}$ and $B = \{5,9\}$, then A and B are disjointed sets, which can be represented on a Venn diagram as shown next:



Example 2

If $X = \{a,b,c,d\}$ and $Y = \{g,f,k\}$, then X and Y are disjointed sets which can be represented on a Venn diagram as

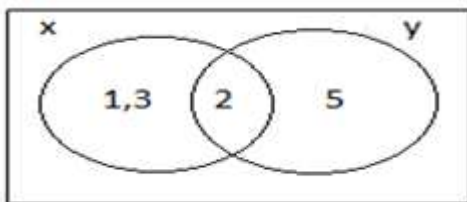


Jointed Sets:

- These are two sets which contain one or more elements in common.

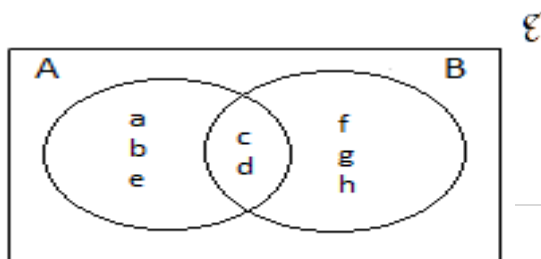
Example (1):

If $x = \{1, 2, 3\}$ and $Y = \{2, 5\}$, then X and Y are jointed sets, which can be represented on a Venn diagram as:



- Example 2.

If $A = \{a,b,c,d,e\}$ and $B = \{c,d,f,g,h\}$, then A and B are jointed sets, which can be illustrated on a Venn diagram as:



8) The Universal Set:

- This is represented by the symbol or U. It is a set which is always bigger than the set under consideration. For example if our set under consideration is {Fantis}, then any of the following sets can be Chosen as the universal set: $A = \{\text{Akans}\}$, $B = \{\text{Ghanaians}\}$ and $C = \{\text{Africans}\}$.
- Also if our set under consideration is $\{1,2\}$, then any of the following sets can be chosen as the universal set: $A = \{1,2,3\}$, $B = \{1,2,3,4,5\}$, $C = \{\text{Integers}\}$ and $D = \{\text{Whole Numbers}\}$.

9) Subset:

- If $A = \{1,2,3\}$ and $B = \{2,3\}$, then we say that B is a subset of A, which is written as $B \subset A$ or $A \supset B$. For B to be a subset of A, then (i). All the members of B must also be members of A.

(ii). The set A must contain one or more elements which are not found in B.

Example 1.

If $Z = \{1,2,3,4,5\}$ and $W = \{2,5\}$, then $W \subset Z$.

Example 2.

- If $Y = \{a,b,c,d,e\}$ and $x = \{b,d,e\}$, then $X \subset Y$.

- But if $A = \{1,3,7\}$ and $B = \{3,8\}$, then B is not a subset of A, which is written as $B \not\subset A$ or $A \not\supset B$.

– This is due to the fact that 8 is not a member of A.

– Also if $X = \{1,2,3,4,5\}$ and $Y = \{1,2,9,8\}$ then $Y \not\subset X$, since 9 and 8 are not members of X.

Q1. The universal set U is given as $U = \{1,2,3,4,5,6,7,8,9,10\}$. Determine which of the following sets are subsets of the given universal set:

i. $A = \{1,2,3\}$ ii. $B = \{5,6,10\}$ iii. $C = \{8,10,44,12\}$ iv. $D = \{1,8,20\}$ v. $E = \{2,3,15\}$

NB: Before a set A can be a subset of a set B, then.

i. All the members of A must also be members of B.
items, which are not found in A.

ii. The set B must contain one or more

Soln

i. $A = \{1,2,3\}$. Since all the members of A are also found in the given universal set, then A is a subset of the given universal set.

ii. $B = \{5,6,10\}$. Since all the members of B are also found in the given universal set, then B is a subset of the given universal set.

iii. $C = \{8,10,11,12\}$. Since some of the members of C {ie 11 and 12} cannot be found in the given universal set, then C is not a subset of the given universal set.

iv. $D = \{1, 8, 20\}$. Since 20 cannot be found in the given universal set, then D is not a subset of the given universal set.

Q2. You are given the set $M = \{a, b, c, d, e\}$. Determine which of these sets are subsets of:

i. $X = \{a, b, c\}$

ii. $Y = \{a, e, k, m\}$

iii. $N = \{c, d, e\}$

iv. $K = \{a, e, g\}$

soln

i. $X = \{a, b, c\}$. Since all the members of the set X are also members of the set M, then X is a subset of M.

ii. $Y = \{a, e, k, m\}$. Because some of the members of Y are not members of M, then Y is not a subset of M.

iii. $N = \{c, d, e\}$. Since all the members of N are also members of M, then N is a subset of M.

(iv) K is not a subset of M, because g is not found in M but found in K.

Q3. If $Q = \{1, 2, 3, 4\}$, write down all the possible subsets of Q.

Soln

The possible subsets are: $\{1, 2, 3\}$, $\{1, 2\}$, $\{2, 4\}$, $\{2, 3, 4\}$, $\{4, 1\}$, $\{3, 1\}$, $\{3, 2\}$, and $\{3, 4\}$.

Q4. If $P = \{1, 2, 3, 4\}$, write down all the subsets of P having exactly two elements.

Soln

$\{1, 2\}$, $\{1, 3\}$, $\{1, 4\}$, $\{2, 3\}$, $\{2, 4\}$ and $\{3, 4\}$.